ISSN 1840-4855
e-ISSN 2233-0046

Original scientific article
http://dx.doi.org/10.70102/afts.2024.1631.168

ANALYSIS OF FLYASH AGGREGATE BEHAVIOR IN GEOPOLYMER CONCRETE BEAMS USING METHOD OF INITIAL FUNCTIONS (MATHEMATICAL PROGRAMMING)

Pushpendra Singh Palash[1*], Priyanka Dhurvey[2]

[1]Research Scholar, Department of Civil Engineering, MANIT-Bhopal, Madhya Pradesh.*
e-mail: palash.rgpv@gmail.com, orcid: https://orcid.org/0000-0002-9074-1683
[2]Assistant Professor, Department of Civil Engineering, MANIT-Bhopal, Madhya Pradesh.
e-mail: pdhurvey@gmail.com, orcid: https://orcid.org/0000-0003-4840-7144

SUMMARY

The stipulated performance of flyash aggregates in geopolymer concrete beams (composite beam) has been explained using the Method of Initial Functions (MIF) via mathematical programming. This research has specifically focused on understanding its strength and durability characteristics. Geopolymer concrete is now being evaluated as a successful alternative in terms of sustainability in the construction sector and has used flyash, an industrial by-product, for several years as a major binder. The major theme adopted in the present research is concentrated on the mechanical and structural behavior of geopolymer concrete beams partially replaced by flyash aggregates for civil engineering applications. The present paper focuses on using the Method of Initial Functions to model and analyze beam behavior subjected to various loading conditions within a strong mathematical programming approach. In the current study, an explanatory analysis of the flexural strength, load-deflection characteristics, and crack opening profile is conducted without constructing a beam specimen by using MIF.

Key words: *lightweight geopolymer concrete, flyash aggregate, method of initial functions beams (MIF).*

Received: July 22, 2024; Revised: August 30, 2024; Accepted: September 18, 2024; Published: October 30, 2024

INTRODUCTION

Geopolymer concrete has emerged as an eco-friendly alternative to conventional Portland cement concrete, offering a reduced carbon footprint and enhanced mechanical properties **[1].** However, a key challenge in geopolymer concrete technology is the development of lightweight versions with improved strength characteristics. Lightweight concrete provides benefits such as better thermal insulation, reduced dead load, and increased design flexibility, making it highly desirable for various construction applications **[2].** The aim of this study is to enhance the strength of lightweight geopolymer concrete by including sintered flyash aggregates. Over the past century, concrete has become an increasingly important construction material, with lightweight concrete (LWC) being successfully utilized in numerous projects, including long-span bridges, high-rise frames, and offshore structures **[3], [4], [5], [6].**

Method of Initial Functions (MIF)

The Method of Initial Functions (MIF) was introduced in 1951 by Malieev. This innovative approach proposed the use of concrete beams incorporating aggregates of different sizes to enhance packing density. Vlasov further developed it in 1955. MIF enables modeling of beam properties without physical construction by deriving governing flexure equations without assumptions about physical behavior, drawing from elasticity theory for precise results across stress-strain states. Recently, MIF has seen wide use, including solving 3D elasticity equations for spherical and cylindrical shells via Taylor series expansions to compute stresses and displacements. Crucially, MIF allows evaluation of the unique composition challenges of laminated beams, like those with flyash aggregate in geopolymer concrete, through mathematical modeling techniques.

Notations [11, 14]

L	Beam length
H	Beam depth
B	Beam width
D	Density of lightweight geopolymer concrete
E	Young’s modulus of elasticity of lightweight geopolymer concrete
F	Compressive strength of lightweight geopolymer concrete
G	Shear modulus of elasticity of lightweight geopolymer concrete
M	Poisson’s ratio
V	vertical displacement
Rx	bending stress

LITERATURE REVIEW

Literature Review on Method of Initial Functions (MIF)

The Method of Initial Functions (MIF) is used for the analysis of beams under symmetric central loading and uniform loading [12]. For the analysis of free vibration in rectangular beams of any depth, the MIF is employed. Frequency values are calculated using the Timoshenko beam theory, presenting the analysis for different values of Poisson's ratio [13]. MIF is used to create governing equations for composite laminated deep beams. The proposed beam theory can be applied to a wide range of beam depths. The analysis of modified deep beams is conducted using the Method of Initial Functions, and the outcomes are compared with existing theory [7, 3]. This approach expands partial differential equations of stress and deflections using Maclaurin's series along the thickness coordinate. Solutions are expressed as unknown initial functions on a reference plane, as proposed [5]. The method incorporates two-dimensional elasticity equations, as outlined in. It has been employed to examine beams under various loading conditions and end constraints, as demonstrated in [13]. MIF has found applications in developing theories for thick laminated composite rectangular plates. Researchers extended this work to composite laminated deep beams, comparing results with existing theories. The method's versatility was further demonstrated in its application to brick-filled reinforced concrete beams [11]. Subsequent studies explored depth-span ratios and the influence of elastic properties on beam behavior [11]. Researchers highlighted the challenges in accurately predicting stress and deflection distributions in laminated beams. Addressing this, they developed a technique to equate unbonded prestressed tendon areas to non-prestressed steel, facilitating the study of composite sandwich structures. Their method effectively predicts deflection up to the point of non-prestressed steel yielding, as validated against experimental data from beams with external unbonded steel and aramid fiber reinforced polymer tendons. Other researchers investigated three-point beam bending using Timoshenko beam theory. Their analytical solutions for deflection, horizontal displacement, and cross-section rotation were then compared to classical Timoshenko beam theory.

METHODOLOGY AND RESULTS

Mix Design (as per IS:10262-2019) of M-50 Grade Concrete mix & Geopolymer Concrete

Design of M-50 grade concrete mix following the guidelines of IS: 10262-2009, Concrete Mix Proportioning. In Geopolymer concrete mix Sodium hydroxide (solid) and sodium metasilicate (glass water) form. Sodium hydroxide and sodium metasilicate is 1:2 ratio with flyash aggregate in table 1[12]**.**

Table 1. Material requirement for GPC block

Designation of Mix	Aggregate		GGBS (kg)	Flyash (kg)	Alkaline solution	Sodium Hydroxide	Sodium hydroxide:
	C.A (kg)	Sand (kg)				Molarity (M)	Sodium Silicate
Flyash aggregate	552	552	373.07	33	243.64	12.5 M	2

Elastic Modulus

For the ambient cured geopolymer concrete, the maximum compressive strength attained was approximately 55 MPa. The linear equation shown below, with the highest R-squared value of 0.642, was confirmed to be appropriate [8]:

Ec = 4 X10-6X (γc)2.66 X (fc')0.5………………………………(ii)[10]

Where Ec and fc' are measured in N/mm^2. The proposed linear equation (Equation (ii)) with an R2 of 0.642 provides a reasonable fit to the experimental data for ambient cured GPC.

Poisson's Ratio

Compared to other geopolymer concrete properties, fewer test results were available for Poisson's ratio, with 99 total data samples collected for this study. No existing equations were found for predicting the Poisson's ratio of geopolymer concrete. When compared to other mechanical parameters, Poisson's ratio exhibited a relatively weak correlation with compressive strength. To address this issue, a correlation was developed between compressive strength and the normalized Poisson's ratio (υ/fc').

The analysis correlating Poisson's ratio with compressive strength revealed a significant coefficient of determination. Through regression analysis, the following finalized equation was derived for predicting the Poisson's ratio of geopolymer concrete:

μ=0.2324/((Fc)^0.093) ……………………………...(iii)

The following beam dimension values were selected for the specified problem.

H = 400mm; L = 3000mm and b = 150mm.

The boundary condition of simply supported edges is given by

X = Y = v = 0; at x = 0 and x = 1.

A uniformly distributed load of 20 N/mm^2 is applied to the surface of a simply supported beam.

Calculation of Compressive Strength, E, μ and G

In table 2 show compressive strength of geopolymer concrete cubes was evaluated per IS516:1959, weighing each mold to determine lightweight geopolymer concrete density. A 3000KN capacity testing machine assessed cube strengths for mixes a (normal aggregate), B, C, D, E and F containing varying (20, 40, 60, 80, and 100%) amounts of sintered flyash aggregate replacement of natural aggregate.

Table 2: Compressive strength, calculation of E, μ and G

Type	% of FAA	Density/m^3	compressive strength	calculation of E	Calculation of μ	Calculation of value G
A	0	2430.33	76.7407	35517.5	0.1552	20515.1
B	20	2314.5	74.8148	30797.2	0.1556	17794.3
C	40	2198.67	72.8889	26517.9	0.156	15326.8
D	60	2082.85	71.8519	22798.6	0.1562	13179.5
E	80	1967.02	70.6667	19417.7	0.1564	11227.4
F	100	1851.19	69.6296	16401.3	0.1566	9485.04

Composite Beam Analysis using MIF and Bending Theory

Conventional theories cannot adequately examine lightweight geopolymer concrete beams made with composite materials like flyash aggregates replacing coarse aggregates. Therefore, this study employs MIF to theoretically analyze stresses and displacements in these beams based solely on their elastic properties and theoretical loads, without requiring experimental flexural testing. The theoretical load P0 was determined from the 28-day compressive strength following limit state design principles for beams [9].

For the MIF analysis, a point load applied to the beam's top surface was represented as a sine series expansion:

$$P(x) = \frac{\mathrm{P0}}{l}_0 + \sum_{n=1}^{\infty}\left(\sin\frac{\pi x}{l}\right)\text{……... (i)}$$

At every 80 mm depth of the beam, the stresses and displacement are calculated and compared with theoretical results..

Analytical Findings and Discussion

The table below presents the stresses and displacements calculated analytically using the Method of Initial Functions (MIF) and classical bending theory. The beams are analyzed based on their elastic properties and theoretical loads, without relying on experimental analyses. Comparisons are made between the MIF and bending theory results for different percentages of coarse lightweight aggregates incorporated in the beam compositions. The graphs for each percentage of replacement along with the depth of the beam are explained below in addition with these findings.

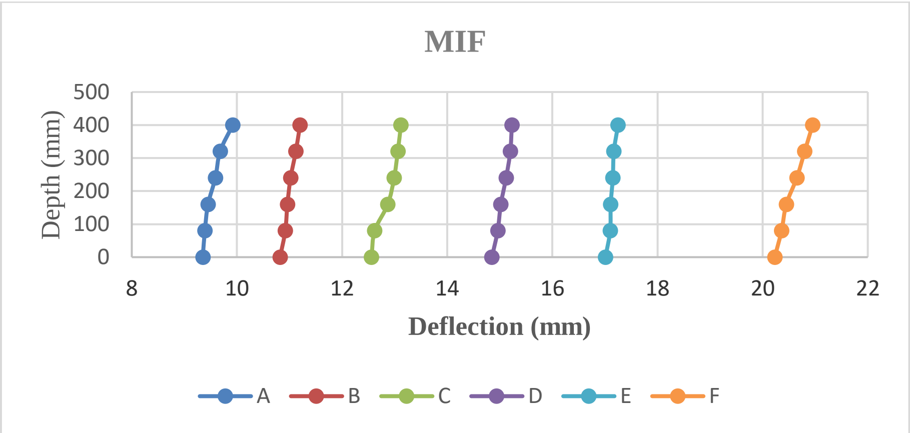

Figure 1. Deflection vs. depth of beam by MIF

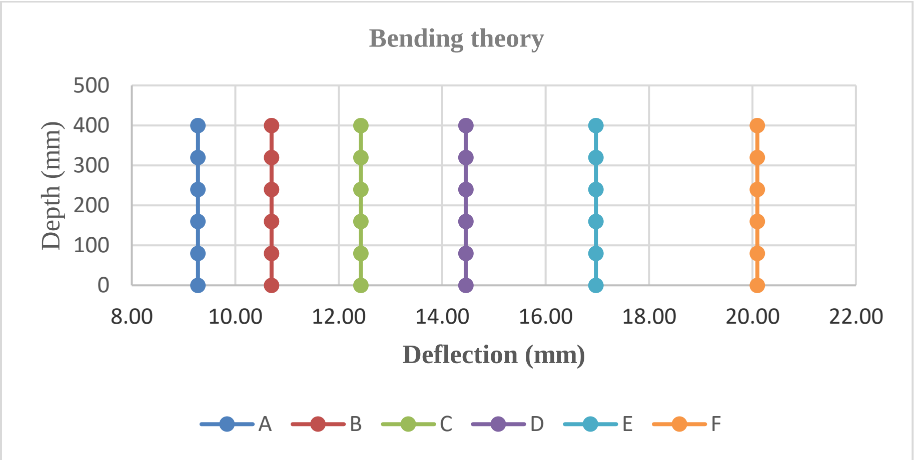

Figure 2. Deflection Vs depth of beam by bending theory

The displacement variation across the beam's depth is depicted in Figure 1&2. Across the depth, the displacement variation (v) is essentially linear. Displacement depends upon the value of modulus of elasticity of that material. When compared to the results of the bending theory in Figure 2, the displacement results from MIF reveal in Figure 1 the exact displacement, that is, displacement at different depths of the beam has been illustrated below.

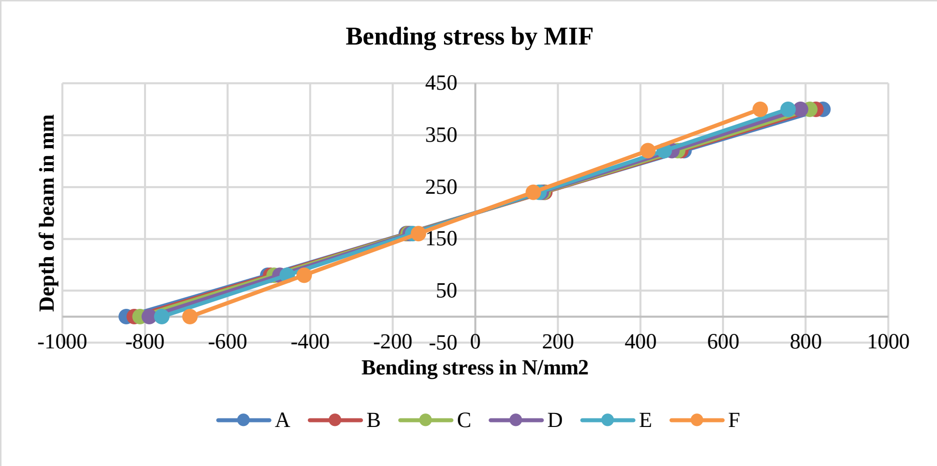

Figure 3(a). Bending stress vs beam depth as calculated using MIF

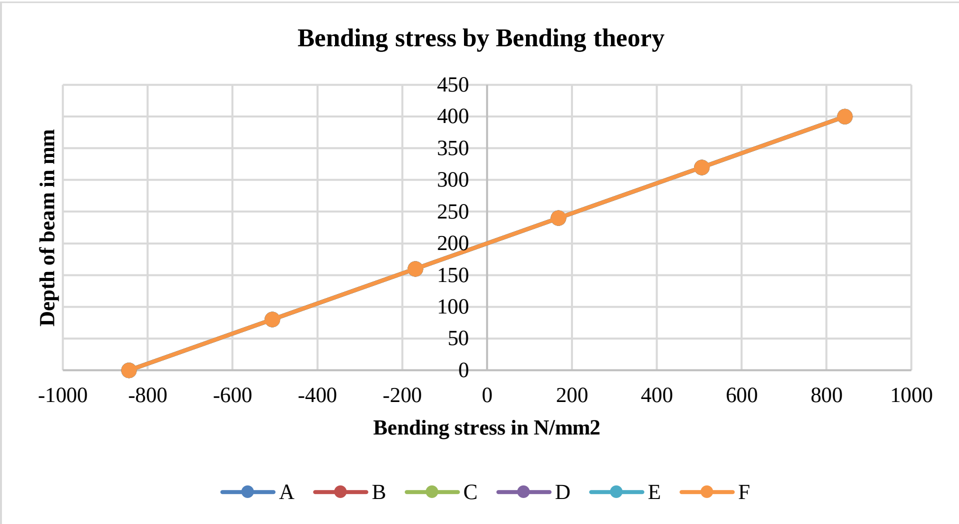

Figure 3(b). Using bending theory calculated a relation between bending stress vs. beam depth

For various percentages of flyash aggregate replacement, the variation in bending stress over the beam depth is depicted in Figures 3(a) and 3(b). Deflection is dependent on the material's elastic modulus value. The graphs demonstrate that the bending stress results from the Method of Initial Functions (MIF) and bending theory are nearly identical. According to the MIF findings, the bending stress exhibits a non-zero value. Bending theory dictates that the bending stress should be zero at the neutral axis and maximum at the top fibers. By substituting different percentages of coarse aggregates with varying sizes of flyash aggregates in the geopolymer concrete, the bending stress decreases as the density increases.

MIF and Bending Theory Comparison

Figures 4 and 5 present the displacement results for the B geopolymer concrete beam containing both flyash and normal aggregates. The displacement predicted by the Method of Initial Functions (MIF) is seen to be in close agreement with the displacement from classical bending theory. While the MIF displacement curve exhibits an irregular shape, the bending theory displacement plot is a straight line. The MIF displacements demonstrate near-identical or similar values to bending theory, though varying slightly at different depths across the beam section. In table 3 a maximum -6.46% variation in displacement is observed at the 0 mm depth (beam bottom), reducing to -4.08% variation at the 400 mm depth (beam top) when compared to bending theory predictions.

Table 3. Comparison of displacement

Depth (mm)	Deflection (mm)		
	MIF	Bending Theory	% Change
0	9.28	9.92	-6.46
80	10.7	11.2	-4.45
160	12.43	13.12	-5.27
240	14.46	15.23	-5.08
320	16.97	17.25	-1.6
400	20.1	20.95	-4.08

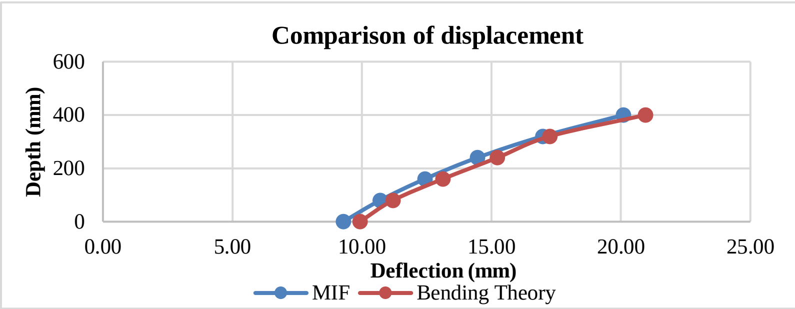

Figure 4. Comparison of displacement by MIF and bending theory

In the case of MIF displacement varies along the depth of the beam and in case of bending theory it is uniform throughout the depth in table 4.

Table 4. Comparison of bending stress

Depth (mm)	Stress σx (N/mm^2)		
	MIF	Bending Theory	% Change
0	841.25	843.75	-0.30
80	824.425	843.75	-2.29
160	810.1238	843.75	-3.99
240	787.41	843.75	-6.68
320	757.125	843.75	-10.27
400	689.825	843.75	-18.24

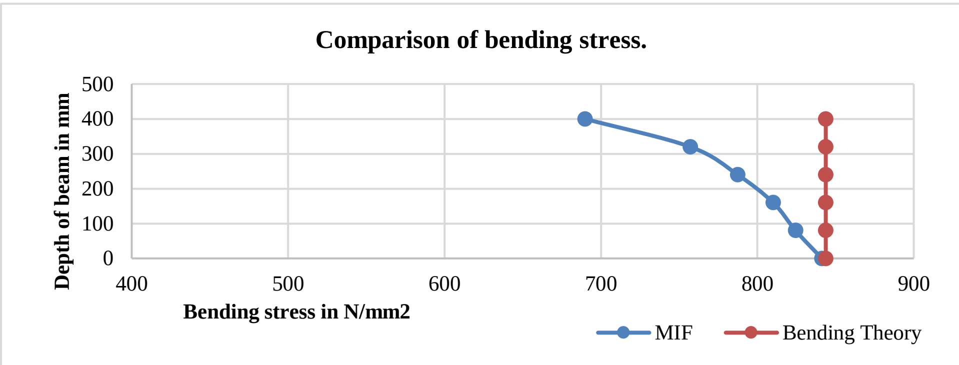

Figure 5. Comparison of bending stress

The figures illustrate that the bending stresses calculated by the Method of Initial Functions (MIF) are nearly identical to those obtained from classical bending theory formulations. MIF analytically computes the precise variations in stresses and displacements along the beam depth by considering the elastic material properties. In contrast, bending theory relies on simplifying assumptions and section characteristics. The stress and deflection results from MIF show very close agreement with bending theory findings as the depth varies across the beam section. However, slight variations are observed, with MIF predicting -0.30% difference in stress at the beam bottom and -18.24% difference at the top compared to bending theory.

CONCLUSIONS

- Natural coarse aggregates can be conserved by effectively utilizing flyash aggregate replacement by natural aggregate in geopolymer concrete without compromising mechanical strength.
- This study provides evidence supporting the replacement of conventional concrete with geopolymer concrete for structural component design.
- The Method of Initial Functions (MIF) model accurately determines stresses and displacements, evaluating them across the beam depth based on the elastic properties of lightweight geopolymer concrete material.
- Unlike bending theory formulas, MIF uses an elastic theory approach, yet its results closely resemble bending theory, validating its suitability for analyzing lightweight beams.
- MIF provided nearly exact values for displacements and stresses of lightweight beams from top to bottom depth.
- The conclusions highlight the novelty of using flyash aggregates in beams and the efficacy of the MIF model for analyzing stresses and displacements in lightweight beams.

CONFLICTS OF INTEREST

The authors declare that they have no competing interests.

REFERENCES

[1] Ahmaruzzaman M. A review on the utilization of fly ash. Progress in energy and combustion science. 2010 Jun 1;36(3):327-363.
[2] Aslam M, Shafigh P, Jumaat MZ. Oil-palm by-products as lightweight aggregate in concrete mixture: a review. Journal of cleaner production. 2016 Jul 10;126:56-73.
[3] Alengaram UJ, Mahmud H, Jumaat MZ. Enhancement and prediction of modulus of elasticity of palm kernel shell concrete. Materials & Design. 2011 Apr 1;32(4):2143-2148.
[4] ACI Committee 213. Guide for structural lightweight-aggregate concrete (ACI 213R-03). Farmington Hills, MI: American Concrete Institute. 2003.
[5] Bui DD, Hu J, Stroeven P. Particle size effect on the strength of rice husk ash blended gap-graded Portland cement concrete. Cement and concrete composites. 2005 Mar 1;27(3):357-366.
[6] Chandra S, Berntsson L. Lightweight aggregate concrete. Elsevier; 2002 Oct 28.
[7] Delhi N. Report on Fly Ash Generation at Coal/Lignite Based Thermal Power Stations and its Utilization in the Country for the year 2016-17. Cent. Electr. Auth. 2017:1-63.
[8] Davidovits J. Geopolymer chemistry and applications. 4th ed. Saint-Quentin, France: Institut Géopolymère, 2015.
[9] Lin CI, Sung WP, Chen KS. Study on management method for fireproof duration of constructional material. Journal of Information and Optimization Sciences. 2004 May 1;25(2):349-360.
[10] Iyengar KS, Chandrashekhara K, Sebastian VK. Thick rectangular beams. Journal of the Engineering Mechanics Division. 1974 Dec;100(6):1277-1282.
[11] Sundara Raja Iyengar KT, Raman PV. Free vibration of rectangular beams of arbitrary depth. Acta Mechanica. 1979;32(4):249-259.
[12] Patel R, Dubey SK, Pathak KK. Analysis of infilled beams using method of initial functions and comparison with FEM. Engineering Science and Technology, an International Journal. 2014 Sep 1;17(3):158-164.
[13] Cui Y, Gao K, Zhang P. Experimental and statistical study on mechanical characteristics of geopolymer concrete. Materials. 2020 Apr 2;13(7):1651. https://doi.org/10.3390/ma13071651
[14] Asutkar P, Shinde SB, Patel R. Study on the behaviour of rubber aggregates concrete beams using analytical approach. Engineering Science and Technology, an International Journal. 2017 Feb 1;20(1):151-159.