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LEXICOGRAPHIC BI-OBJECTIVE OPTIMIZATION FOR FRACTIONAL ORDER CONTROLLER TUNING IN GRID-CONNECTED STATCOM APPLICATIONS

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SUMMARY

Static Synchronous Compensators (STATCOMs) require precise controller tuning to maintain voltage stability in modern power grids. Fractional Order Proportional-Integral (FOPI) controllers offer enhanced flexibility over integer-order designs, yet existing tuning methods rely on weighted-sum formulations that conflate conflicting objectives through arbitrary weight selection, overlooking the inherent priority structure of power system stability requirements. This paper proposes a lexicographic bi-objective optimization framework that enforces strict priority ordering without weight selection, implemented through two formulations: a Lexicographic ITAE approach evaluated across five meta-heuristic algorithms with hard penalty constraints, and a Lexicographic PSO approach employing a dual-criteria ranking mechanism with a normalized sacrifice metric. Both were verified using an 11 kV cascaded H-bridge STATCOM in Simulink. Observations showed that there was a "performance inversion" effect where algorithms that provided the least cost gave oscillating or shifted voltage profiles, whereas those that were penalized resulted in improved physical performance. Obsessive settling time priority attained 0.25 s; however, it resulted in the steady-state shift by 140 V for an 82 V target voltage. Random sampling showed that only 9% of solutions gave a stable response. Between the two algorithms, Lex-ITAE with HHO performed better as it gave a smooth voltage profile close to the target 82 V despite having higher costs numerically.

Keywords: *lexicographic optimization, FOPI controller, STATCOM, particle Swarm optimization, multi-objective optimization, voltage stability.*

INTRODUCTION

Voltage stability and power quality problems in modern power networks have become major concerns due to incorporation of renewable resources and high demand of load [1][2]. STATCOM devices are used to address such problems as reactive power support, and its operation depends on the internal control strategy [3][4]. FoPID regulators have advantages over their integer-order counterparts as use extra tuning parameters (λ, μ). However, choosing appropriate values of these parameters can be difficult due to conflicting requirements on minimum overshoot and fast response time [5][6]. In this context, meta-heuristic algorithms such as PSO, GWO, and GA are widely applied to solve the problem via a weighted-sum approach [9].

In this case, there is an obvious problem, as tuning parameters are selected based on the trial-and-error method [12]. The reason for this drawback lies in the fact that the weighted-sum approach ignores the hierarchical character of the power system control. As a result, the assumption is made that the conflict between such safety-critical requirements as minimal overshoot and secondary measures like steady -state error can be compromised with equal importance [11]. Thus, a solution to the tuning problem, taking into account the hierarchy of priorities, is required.

In this paper, explored lexicographic tuning of FOPI controllers for STATCOMs, posing the question on how hierarchical optimization would enable to achieve transient stability in the controllers while achieving maximum steady state accuracy. The sacrifice on the main goal is defined through a ranking algorithm using a normalized sacrifice function. There are two approaches explored in this paper, namely: Lex-ITAE, where ITAE is used as both transient and steady state objectives in five meta -heuristics under hard constraint penalty, and Lex-PSO, which uses the time to settle and tracking RMSE as objective functions in PSO to perform bi-objective trade-off analysis based on the proposed ranking method. Random sampling acts as the baseline algorithm for search space difficulty. Section 2 reviews related work; Section 3 develops the framework; Section 4 describes the experimental design; Section 5 reports results; Section 6 discusses findings; Section 7 concludes.

LITERATURE SURVEY

STATCOM control has evolved from classical PID tuning [8] to nature-inspired meta-heuristics. GA and bacterial foraging have been applied to STATCOMs in hybrid microgrids [7], grasshopper optimization to D-STATCOMs [13], and squirrel-based searches to induction-generator microgrids [9]. Multi-objective PSO has addressed STATCOM placement under contingencies [14], while PSO-based stability assessment using synchronizing damping torque coefficients has also been demonstrated [15][10]. Fractional-order controllers have gained momentum: whale optimization for FOPI-STATCOM harmonic mitigation [5], FOPI for hybrid renewables [16], Coati-tuned FOPID for multi-area AGC [6], Dandelion-tuned FOPI-PIDA for wind frequency stability [17], and whale-tuned SVC-PI outperforming EP/AIS methods [18]. These studies uniformly rely on weighted-sum or single-objective formulations that conflate competing criteria. Broader control-design work exhibits the same pattern: hybrid whale optimization for AVR PIDD2 [19], hybrid meta-heuristics for PI-PIDA STATCOM [20], marine-predator PIDA, and survey works all identify the weighted-sum assumption as a common methodological limitation. Pareto-based variants [14] compute entire trade-off fronts but do not exploit priority structure.

Lexicographic optimization, rooted in the ε-constraint method and formalized, provides a principled alternative. Augmented and improved ε-Constraint formulations and hierarchical applications in water systems demonstrate engineering viability, supported by foundational multi-objective works. In spite of all its maturity, however, lexicographic formulations have not yet been utilized in the area of tuning controllers of power systems. It is important to stress the fact that none of the existing research works applies the combination of FOPI controllers, meta-heuristic optimization algorithms, and lexicographic comparison among transient stability and steady state precision for STATCOM controllers. All previous works apply weighted sum approaches allowing uncontrollable trading or Pareto approaches overlooking any priority structure. This paper addresses the gap by proposing a dual-criteria ranking within a two-phase ε-constraint framework tailored for population-based FOPI tuning.

METHODOLOGY

These sections establish shared foundations for the ε-constraint extension, two-phase structure, dual-criteria ranking, and STATCOM application. Sections 3.8–3.9 develop the two formulations.

Theoretical Foundations

Multi-objective problems with conflicting objectives are classically handled by NSGA-II or MOEA/D, which approximates the full Pareto front. When objectives possess an inherent priority, computing the full front is inefficient. Given bi-objective $min[f_1(x), f_2(x)]$, the classical ε-The constraint method is (Equation 1)

$$\min_x f_1(x) \quad subject\ to \quad f_2(x) \leq \varepsilon \tag{1}$$

whose solutions are Pareto optimal by the Equivalence Theorem. The present framework extends this in three ways: (i) adaptive tolerance determined endogenously by Phase I; (ii) a dual-criteria ranking that quantifies primary-objective sacrifice; (iii) a two-phase structure for population-based search. Efficiency gains from region reduction are inherent to ε-constraint methods, the contribution is the ranking and its integration with meta-heuristics.

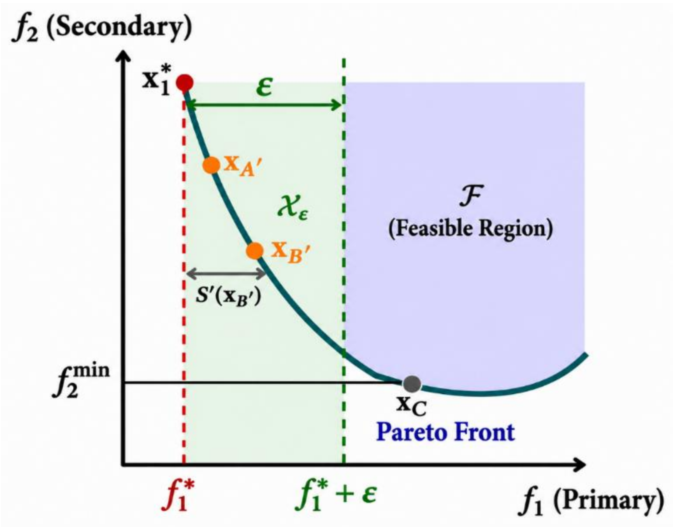


Figure 1. Conceptual illustration of the lexicographic framework in objective space. The tolerance ε defines the reduced feasible region X_ε (green), within which secondary optimization is conducted. $x_A, x_B \in X_\varepsilon$; x_C is excluded despite lower f_2

The figure 1 depicts an example of a lexicographic optimization problem on the feasible set F. The red point denotes the current optimal solution to the first objective function f_1. The green shaded region indicates the ϵ tolerance region in which the secondary objectives may be optimized. Solutions $x_A\ and\ x_B$ belong to the tolerance region and represent acceptable solutions, whereas x_C is the Pareto-optimal solution.

Two-Phase Optimization Framework

Both formulations share the two-phase structure of figure 2: Phase I optimizes f_1 independently; Phase II optimizes f_2 within a tolerance-constrained region anchored by the Phase I result.

Phase I: Primary Objective Optimization

A single-objective meta-heuristic solves (Equation 2)

$$f_1^* = \min_{x \in X} f_1(x) \tag{2}$$

with $x_1^* = arg\, min_{x \in X} f_1(x)$. The relative tolerance constraint (Equation 3)

$$f_1(x) \leq f_1^*(1+\varepsilon), \quad \varepsilon \in [0,1] \tag{3}$$

permits a fractional degradation ε from f_1^*. Phase I quality directly affects Phase II: convergence to a local optimum may exclude valuable trade-offs.

Phase II: Constrained Secondary Optimization

The reduced feasible region is (Equation 4)

$$X_\varepsilon = \{x \in X: f_1(x) \leq f_1^*(1+\varepsilon),\ g_j(x) \leq 0,\ \forall j\} \tag{4}$$

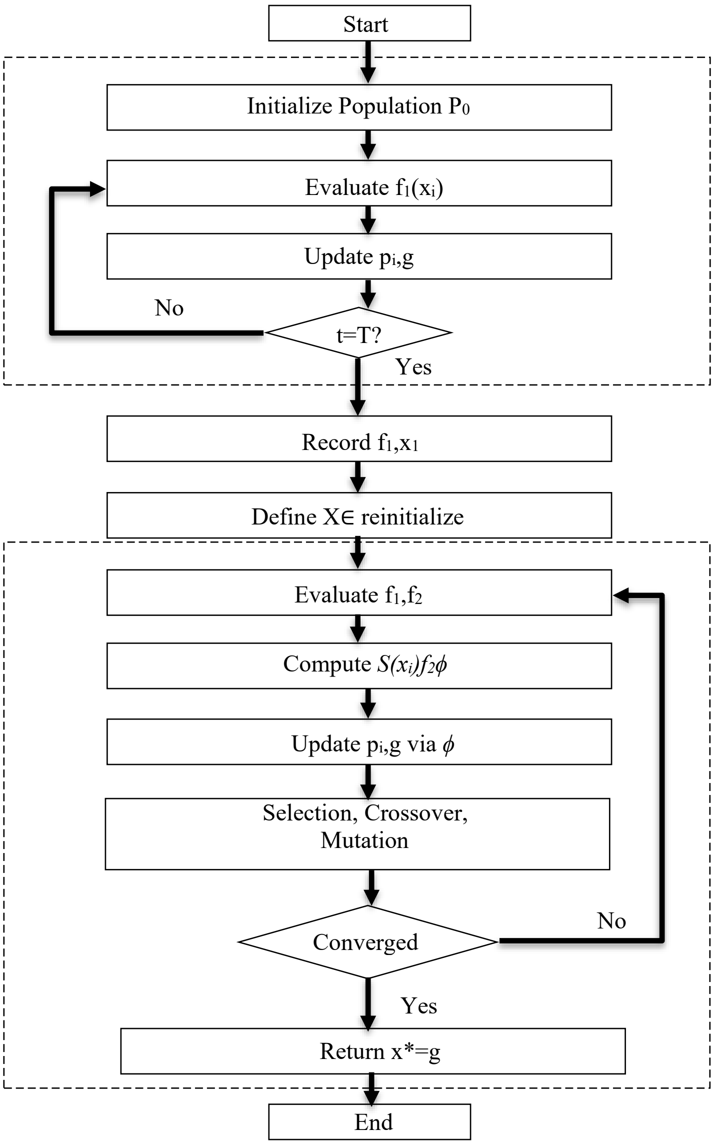


Figure 2. Flowchart of the two-phase lexicographic framework. Phase I yields f_1^*; Phase II optimizes over X_ε using composite fitness Φ. definitions of f_1, f_2, and Φ differ between Lex-ITAE and Lex-PSO.

with volume ratio $\rho = |X_\varepsilon|/|X|$ often much less than unity. A hybrid initialization combines Phase I exploitation with fresh exploration: a fraction $\gamma \in [0.1, 0.3]$ of the population is seeded as (Equation 5)

$$x_i^{(0)} = x_1^* + \sigma \cdot z_i, \quad z_i \sim N(0, I) \tag{5}$$

with $\sigma = 0.1 \cdot \| x_{max} - x_{min} \|$; the remaining $(1-\gamma)N$ are sampled with feasibility bias. Infeasibility is handled via (Equation 6)

$$J_{total}(x) = \Phi(x) + \rho_\varepsilon \cdot max\{0, f_1(x) - f_1^*(1+\varepsilon)\} \tag{6}$$

with $\rho_\varepsilon = 10^{10}$.

Dual-Criteria Ranking Mechanism

The normalized sacrifice metric measures relative primary-objective degradation (Equation 7):

$$S(x_i) = \frac{f_1(x_i) - f_1^*}{f_1^{max} - f_1^*} \tag{7}$$

where f_1^{max} is the current population's maximum. The secondary objective is normalized as (Equation 8)

$$f^{-}{}_2(x_i) = \frac{f_2(x_i) - f_2^{min}}{f_2^{max} - f_2^{min}} \tag{8}$$

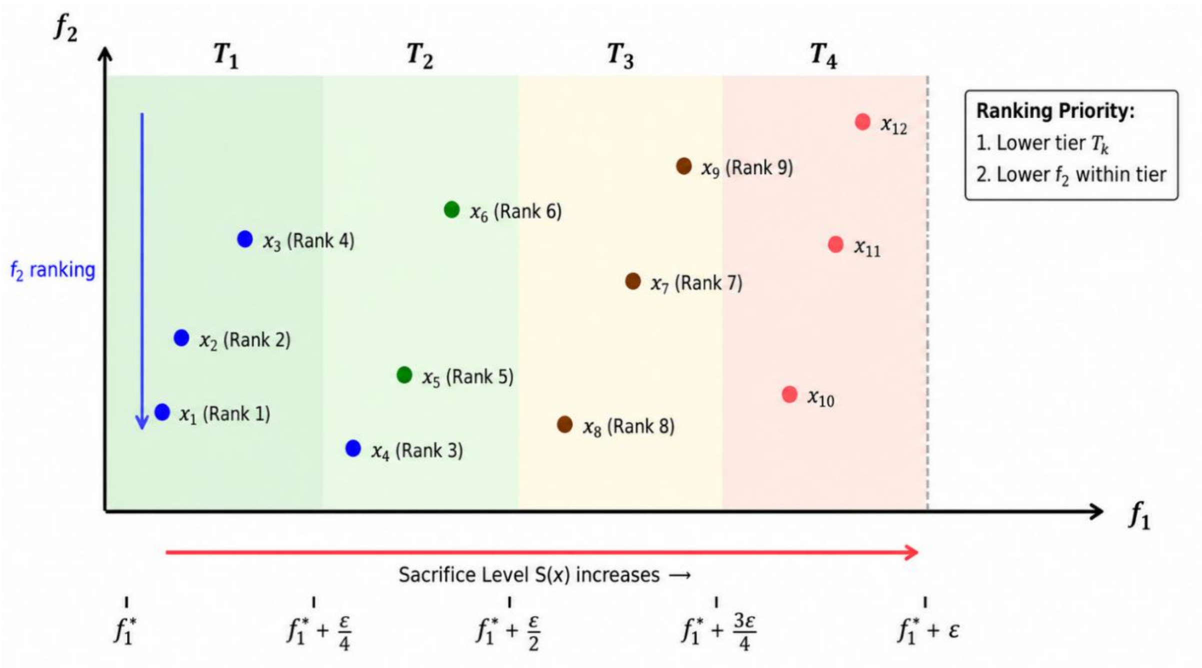


Figure 3. Dual-criteria ranking with sacrifice tiers. The reduced feasible region is partitioned into $K = 4$ tiers by $S(x)$; within each tier, solutions are ranked by f_2 ascending

and the composite fitness combines both (Equation 9):

$$\Phi(x_i) = \alpha \cdot f^{-}{}_2(x_i) + (1-\alpha) \cdot S(x_i) \tag{9}$$

where $\alpha \in [0,1]$ trades emphasis between secondary optimization ($\alpha = 1$) and primary preservation ($\alpha = 0$). Figures 3&4 (a - c) illustrate sacrifice tiers and the effect of α.

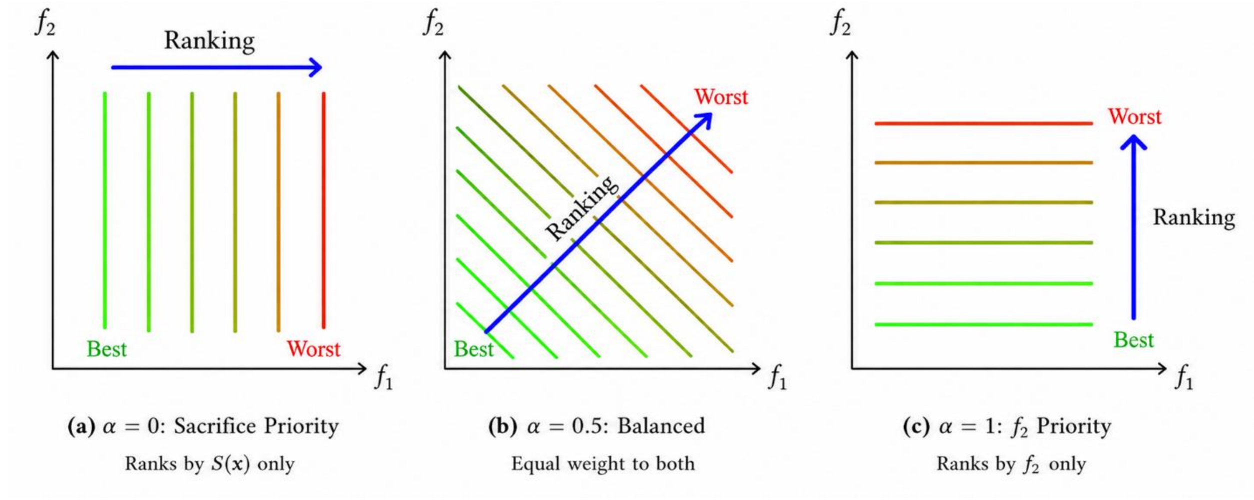


Figure 4(a-c): Effect of α on composite fitness contours and ranking direction. (a) $\alpha = 0$: ranked by sacrifice. (b) $\alpha = 0.5$: balanced. (c) $\alpha = 1$: ranked by f_2

Adaptive tier ranking partitions solutions by sacrifice level (Equation 10):

$$T_k = \left\{x_i : \frac{k-1}{K} \le S(x_i) < \frac{k}{K}\right\}, \quad k = 1, \dots, K \tag{10}$$

Within each tier, solutions are ranked by f_2; the overall rank combines tier and within-tier ordering so lower-sacrifice solutions are always preferred.

Lex-ITAE Algorithm

Algorithm 1 formalises the Lex-ITAE procedure, parameterised by meta-heuristic A (PSO/GWO/HHO/ALO/AHA).

Algorithm 1: Lexicographic ITAE(Lex-ITAE)

```
Input: Meta-heuristicA, population size N, tolerance ε ,max iterations T_max,bounds[LB, UB]
Output: Best FOPI parameters x*
/* Phase I: Transient ITAE Minimization */
Initialize P_0 randomly within [LB, UB];
for t=1toT_max do
    for each x_i ∈ P_t do
        Simulate STATCOM with x_i;
        Detect T transient via Eq. (17);
        Compute f_1^ITAE(x_i) via Eq.(15);
        Apply A's operators based on f_1^ITAE;update P_{t+1};
        J_1* ← f_1^ITAE ; x_1* ← argmin f_1^ITAE ; J_limit ← J_1*(1 + ε);
```

```
/* Phase II: Steady-State ITAE with Hard Penalty */
    Reinitialize P_0:seed γN around x_1^*;sample (1−γ) N randomly;
    for t=1 toT_max do
      for each x_i ∈ P_t do
        Simulate; detect T transient; compute f_1^ITAE ,f_2^ITAE ;
        if f_1^ITAE(x_i) > J_limit then
          J_total(x_i) ← 10^1 0 + 10^3(f_1^ITAE (x_i) − J_limit);
        else
          J_total(x_i) ← f_2^ITAE (x_i);
          Apply A's operators based on J_total; updateP_{t+1};
    eturn x^* ← argmin J_total
```

This algorithm 1 applies the lexicographic approach to optimize FOPI controller tuning in two stages. Stage I seeks to minimize the transient ITAE value using meta-heuristic optimization to find the best starting point for parameters. In stage II, the steady-state ITAE value is minimized under tolerance constraints and the use of hard penalties if constraints are not met.

Formulation II: Lexicographic PSO (Lex-PSO)

Lex-PSO uses physically interpretable objectives settling time and tracking RMSE under the dual-criteria composite fitness *Φ*. It is validated with PSO only to isolate bi-objective trade-off analysis from inter-algorithm variability.

Objective Definitions

The primary objective is the DC-link settling time (Equation 11):

$$f_1(x) = T_s(x) = inf\{t > 0: |y(\tau) - y_{ref}| \le \delta_s |y_{ref}|,\ \forall \tau \ge t\} \tag{11}$$

with $\delta_s = 0.02$, $y_{ref} = 82$ V. The secondary is post-settling tracking RMSE (Equation 12):

$$f_2(x) = \sqrt{\frac{1}{T_{end} - T_s(x)} \int_{T_s(x)}^{T_{end}} e^2(t,x)\, dt} \tag{12}$$

Unlike Lex-ITAE, T_s directly interprets ε as allowable time degradation and RMSE avoids the time-weighting artefact that masks steady-state offset.

Composite Fitness and Constraint Enforcement

Phase II fitness is (Equation 13)

$$\Phi(x_i) = \{\alpha \bar{f}_2(x_i) + (1-\alpha) S(x_i), if\ f_1(x_i) \le f_1^*(1+\varepsilon)\ 10^{10} + 10^3(f_1(x_i) - f_1^*(1+\varepsilon)), otherwise \tag{13}$$

with S, f^{-}_2 Feasible solutions are ranked through normalized Φ—not raw f_2—enabling nuanced trade-offs via α.

PSO Velocity and Position Updates

Velocities follow (Equation 14)

$$v_i(t+1) = \chi[\omega v_i(t) + c_1 r_1(p_i - x_i(t)) + c_2 r_2(g - x_i(t))] \tag{14}$$

with $x_i(t+1) = x_i(t) + v_i(t+1)$, inertia ω damped by ω_{damp}, and personal/global bests p_i, g. Phase I updates use f_1; Phase II uses Φ, so p_i is updated only when $\Phi(x_i^{new}) < \Phi(p_i)$. Parameters: $\omega = 1$, $\omega_{damp} = 0.99$, $c_1 = 1.5$, $c_2 = 2.0$, $T_{max} = 10$, $N = 10$, with boundary clamping.

Lex-PSO Algorithm

Algorithm 2 formalises the procedure. The best compromise across runs is the one with lowest final f_1, consistent with $f_1 \succ f_2$.

Algorithm 2: Lexicographic PSO (Lex-PSO)

```
Input:N, T_max, ε, α, [LB, UB], (ω, c_1, c_2, ω_damp)
Output:Optimal FOPI parameters x*
Initialize swarm: x_i(0) ~ U(LB, UB), v_i(0) = 0;
/* Phase I: Settling Time Minimization */
for t=1 to T_max do
    Simulate; compute f1(xi)=Ts;
    Update p_i, g by f_1;update velocities (Eq.22), positions; clamp; ω ← ω ω_damp;
    f_1* ← f_1(g); x_1* ← g; f_limit ← f_1*(1 + ε);
/* Phase II: Constrained RMSE via Dual-Criteria Ranking */
  Reinitialize: γN elite-seeded;(1 − γ) N random; reset ω;
  for t=1 toT_max do
    Simulate; compute f_1(x_i) = T_s, f_2(x_i) = RMSE; get f_1^max, f_2^min, f_2^max;
    for each particle i do
      if f_1(x_i) > f_limit then
        Φ(x_i) ← 10^10 + 10^3(f_1(x_i) − f_limit);
      else
        Compute S, f_2(Eqs.(8),(9)); Φ ← αf_2 + (1 − α)S;
        Update p_i ifΦ(x_i) < Φ(p_i);
```

15 $g \leftarrow argmin\Phi(p_i); update\ velocities, positions; clamp; \omega \leftarrow \omega\omega_{damp};$

16 return $x^* \leftarrow g$

Two phase PSO has been used in this algorithm 2 to design the controller. In the first phase, the algorithm tries to minimize settling time, adjusting particle position and velocity accordingly, while in the second phase, it uses multi-objective ranking based on RMSE under constraint satisfaction to optimize the steady state performance.

EXPERIMENTAL DESIGN

Three experiments evaluate the framework (Table 1). Lex-ITAE and Lex-PSO this section specifies the simulation environment, algorithm configuration, and protocol.

Table 1. Experimental protocol overview

Exp.	Name	Algorithm(s)	Objectives (f_1 / f_2)	Runs	Ref.
I	Lex-ITAE	PSO, GWO, HHO, ALO, AHA	$ITAE_{trans}$ / $ITAE_{ss}$	5 × 5	Sec. 3.8
II	Lex-PSO	PSO only	T_s / RMSE	5	Sec. 3.9
III	Baseline	Uniform random sampling	T_s / RMSE	1	—

Simulation Environment

Implementation is in MATLAB/Simulink with variable-step solver (ode45). The model represents an 11 kV, 10 MVA distribution grid with a Cascaded H-Bridge STATCOM, comprising a programmable three-phase source, PI-section transmission lines, a shunt-connected VSC coupled via a step-up transformer, and a three-phase fault block for disturbance injection.

The hierarchical dq-frame control uses a PLL for grid synchronization and comprises three cascaded regulation loops: an outer DC-voltage loop in which a FoPID generates the d-axis current reference $I_{d,ref}$ by comparing DC-link capacitor voltages to their setpoints; a middle reactive-power loop in which a FoPID processes the error between measured and reference reactive power to generate $I_{q,ref}$; and an inner decoupled current loop employing two FoPIDs to track current references and generate PWM voltage commands.

Algorithm Configuration

Lex-ITAE uses PSO, GWO, HHO, ALO, AHA; Lex-PSO uses PSO only. Shared settings are in table 2; FOPI search bounds in table 3.

Table 2. Unified algorithm configuration

Parameter	Value
Population size (N)	10
Max iterations (T_{max})	10 per phase
Independent runs (n_{runs})	5
Decision variables (n)	15
Lexicographic tolerance (ε)	0.05 (5%)
PSO	$\omega = 1$, $\omega_{damp} = 0.99$, $c_1 = 1.5$, $c_2 = 2.0$
GWO	Linearly decreasing a from 2 to 0
HHO	Non-linearly decreasing energy parameter
ALO	Roulette wheel selection
AHA	Migration probability = $1/(2N)$

Table 3. FOPI controller search bounds

Parameter	Symbol	Lower	Upper
Proportional gain	K_p	0	50
Integral gain	K_i	0	50
Fractional order	λ	0.001	0.999

Evaluation Metrics

ITAE (Integral of Time-weighted Absolute Error) for transient and steady-state performance (Equation 15):

$$ITAE = \int_0^T t \cdot | e(t) | \, dt \tag{15}$$

Settling Time (T_s) is usually expressed mathematically as the time at which the system output $y(t)$enters and remains within a specified tolerance band around the reference y_{ref} (Equation 16):

$$T_s = min \{t \mid | y(t) - y_{ref} | \leq \epsilon \cdot y_{ref},\ \forall\, t' \geq t\} \tag{16}$$

Here, $\epsilon$is the allowed tolerance (for example, 2% → $\epsilon = 0.02$).

RMSE (Root Mean Square Error) – measures post-settling tracking accuracy (Equation 17):

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2} \tag{17}$$

These three metrics capture the core objectives: ITAE for overall system response, settling time for speed, and RMSE for accuracy.

Experimental Protocol

Lex-ITAE (Exp. I): The five methods repeat five times for each run according to Algorithm 1; the criteria are best/worst/average cost and their standard deviations. Lex-PSO (Exp. II): PSO repeats five times for each run according to Algorithm 2; the best balance of the Pareto Front corresponds to the lowest value of f_1, since $f_1 \leq f_1^{*\,(Lex-PSO)}$. Baseline (Exp. III): Randomly selected 100 FoPID controllers based on the uniform distribution from the 15-dimensional space using table 3; those which are stable are compared using $T_s/RMSE$ and $\frac{T_s}{RMSE}, f_1 \leq f_1^{*\,(Lex-PSO)}\,(1+\varepsilon)$.

RESULTS AND ANALYSIS

Experiment I: Lex-ITAE Multi-Algorithm Comparison

Statistical Metrics of Final Cost

In table 4 summarizes performance after five iterations. Particle Swarm Optimization shows the smallest average cost (0.56) with minimum standard deviation (0.86). The average cost for other algorithms ranges between 40.4–80.1, showing binary behavior of either convergence or default to max penalty cost.

Table 4. Statistical cost metrics across 5 runs

Algorithm	Mean Cost	Std Dev	Best Cost	Worst Cost
AHA	4.04×10^1	5.44×10^1	5.20×10^{-1}	1.00×10^2
ALO	4.06×10^1	5.42×10^1	0.00	1.00×10^2
GWO	6.04×10^1	5.42×10^1	2.75×10^{-1}	1.00×10^2
HHO	8.01×10^1	4.44×10^1	7.26×10^{-1}	1.00×10^2
PSO	5.57×10^{-1}	8.60×10^{-1}	0.00	1.96

Affordable cost does not ensure better physical performance: since ITAE in steady state is computed based on integration from settling time to the end of the simulation, delayed settling of controllers leads to reduced duration for computing ITAE, thus producing deceptive values of ITAE. The cost value of 100 represents the failure of the lexicographic constraint, and not necessarily an unstable system physically. Table 5 contains per-run results while table 6 gives runtimes (PSO fastest ~50,510 s; ALO slowest ~71,500 s).

Table 5. Detailed final cost by run

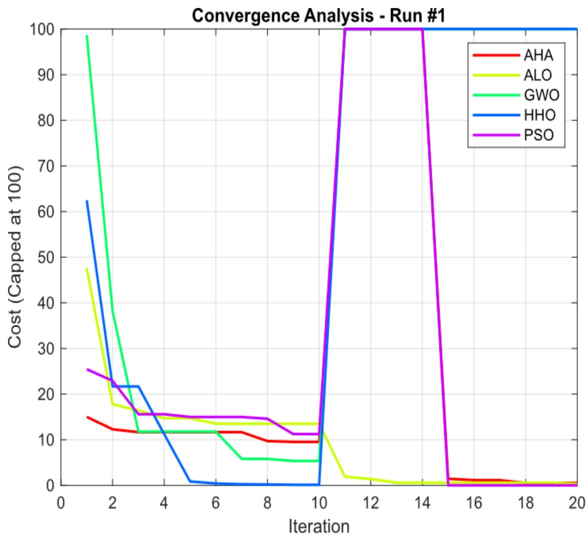
Algorithm	Run 1	Run 2	Run 3	Run 4	Run 5
AHA	0.52	100.00	0.76	100.00	0.58
ALO	0.30	100.00	100.00	0.00	2.70
GWO	100.00	100.00	0.27	100.00	1.71
HHO	100.00	0.73	100.00	100.00	100.00
PSO	0.01	0.00	1.96	0.00	0.81

Table 6. Computational runtime (seconds)

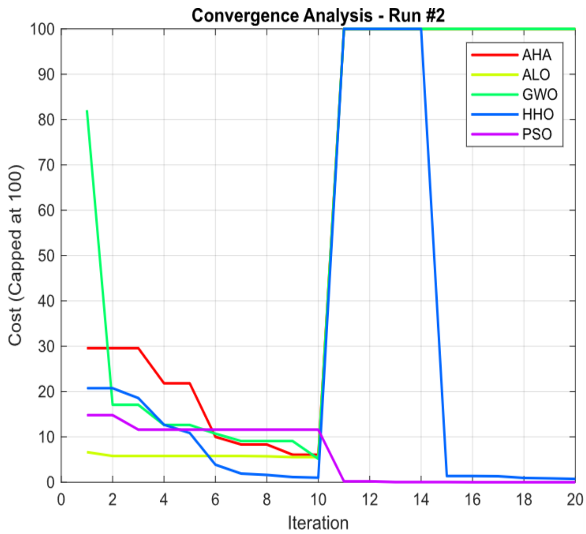
Algorithm	Total (5 Runs)	Mean Run	Fastest Run
AHA	267,773	53,554	51,485
ALO	357,501	71,500	22,398
GWO	253,015	50,603	47,046
HHO	344,401	68,880	61,263
PSO	252,550	50,510	48,641

Convergence Analysis

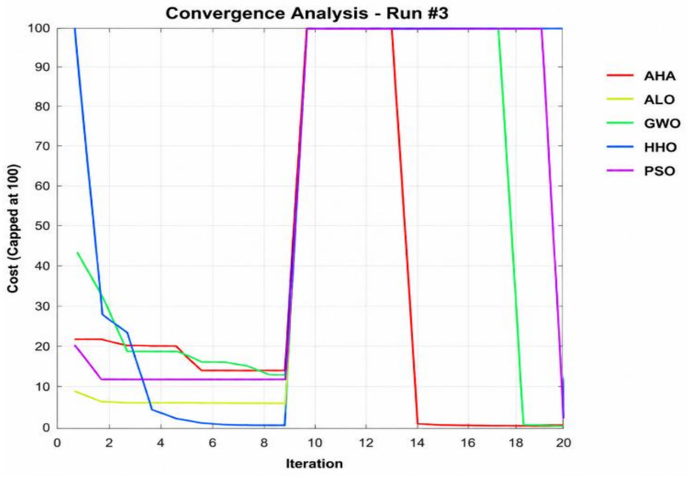
Convergence is achieved in all five iterations in figure 5(a-e). The first ten iterations attempt minimisation of f_1^{ITAE}, while the sharp increase at iteration number 11 signals the start of Phase II where minimisation of f_2^{ITAE} is attempted with the hard constraint. The PSO algorithm manages to consistently return back to zero, this happens at the cost of poor steady-state performance.



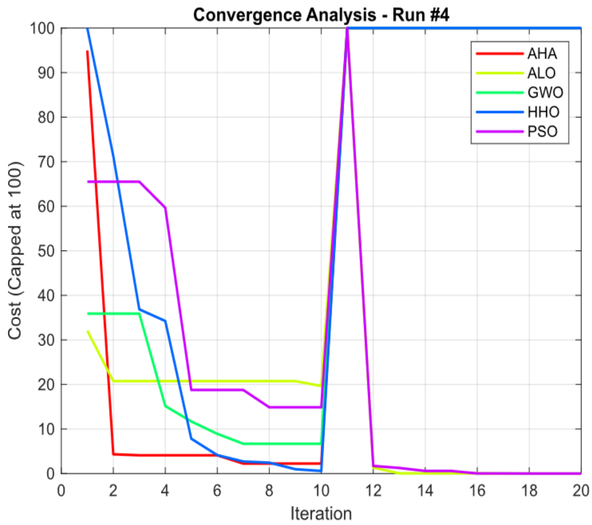
A: Run 1



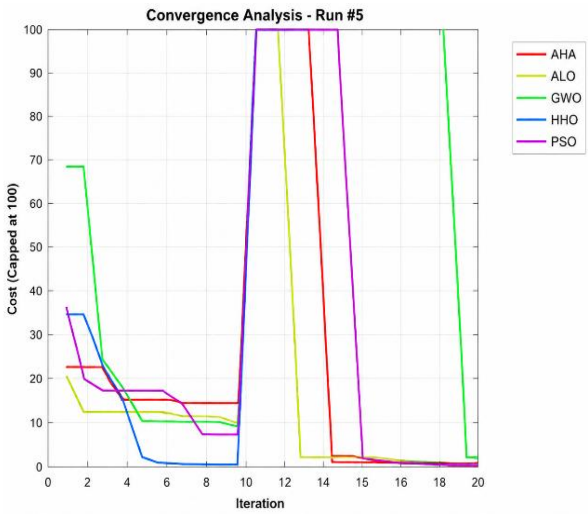
B: Run 2



C: Run 3



D: Run 4

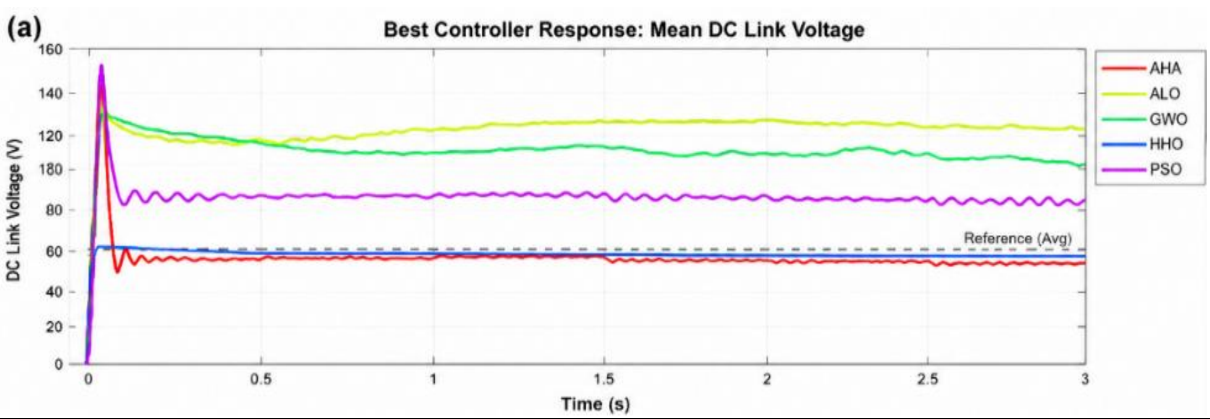


E: Run 5

Figure 5 (a-e): Convergence curves for five independent runs; y-axis capped at 100

Time-Domain Validation

Best parameters obtained from all of the mentioned algorithms were evaluated in Simulink. In the figure for the DC-link (Figure 6a), the vital distinction between the ITAE-based selection and actual regulation is that HHO, having the largest value of ITAE criteria, demonstrated the most precise tracking near 82 V whereas PSO which won according to statistics, had oscillation and offset of ∼105. Figure 6b compares all five algorithms, confirming the performance inversion.



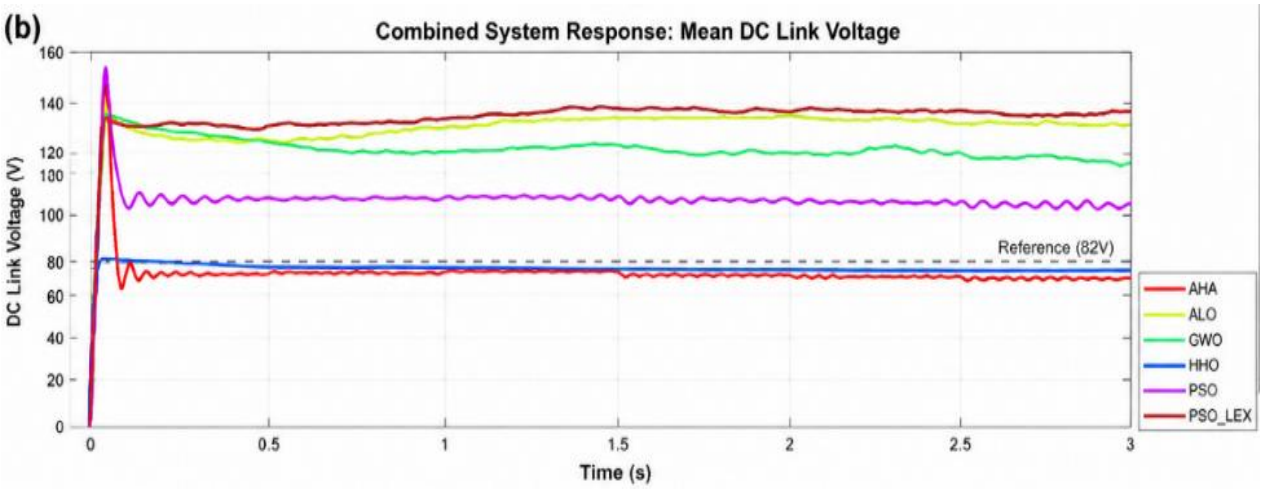


Figure 6(a&b): DC-link voltage response validation and combined time-domain response of optimized controllers showing performance inversion

Experiment II: Lex-PSO Internal Validation

Lex-PSO extracts the ranking based on the two criteria of physically interpretable objectives without manipulating the integration window.

Statistical Analysis

In table 7 lists the results obtained from various runs and reveals that Run 4 is the best because it produces the shortest value of T_s (0.2506 s) with 44% improvement over the second-best run while only marginally increasing the RMSE. Table 8 exhibits the highest value of f_1 SD (0.352) compared.

Table 7. Lex-PSO optimization results (5 runs)

Run	f_1 (Settling Time) [s]	f_2 (RMSE)	Notes
1	0.4456	**56.3266**	Best f_2
2	0.8643	81.5058	Worst
3	1.0718	62.9025	Slowest
4	**0.2506**	57.6243	**Best Compromise**
5	0.9663	60.1482	–

Table 8. Statistical analysis of Lex-PSO objectives

Metric	Min	Max	Mean	Std Dev
f_1 (Settling Time)	0.2506	1.0718	0.7197	0.352
f_2 (RMSE)	56.3266	81.5058	63.7015	10.270

Optimal Decision Variables

In table 9 lists the tuned FoPID parameters from Run 4.

Table 9. Optimal decision variables from run 4

Controller 1 (Current)		Controller 2 (Power)		Controller 3 (DC Voltage)	
Param	**Value**	**Param**	**Value**	**Param**	**Value**
x_1	49.2162	x_6	0.5620	x_{11}	0.0010
x_2	36.7667	x_7	28.2810	x_{12}	0.0228
x_3	0.3165	x_8	10.1207	x_{13}	18.0314
x_4	42.1787	x_9	0.1722	x_{14}	26.1751
x_5	38.0582	x_{10}	13.1753	x_{15}	0.4936

Time-Domain Response

In figure 7 shows Run 4's DC-link voltage: the 0.25 s settling target is met, but the system stabilises at ~ 140 V rather than 82 V, exposing the trade-off of strict settling-time prioritization.

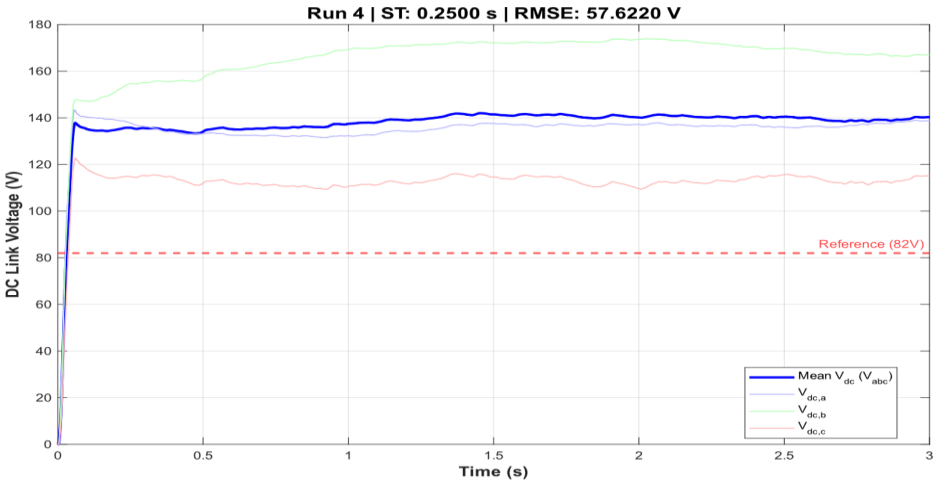


Figure 7. Time-domain V_{dc} from Run 4: fast settling (0.25 s) but significant steady-state offset (~ 140 V vs. 82 V reference)

Phase 1 vs. Phase 2 Comparison

In table 10 quantifies Phase 2 impact: Run 4 improves T_s by 4.9% while minimising RMSE.

Table 10. Impact of phase 2 on primary objective

Run	Phase 1 f_1^*	Phase 2 f_1	Change	Final f_2
1	0.8555	0.4456	−47.9%	56.3266
2	0.8643	0.8643	0.0%	81.5058
3	1.1224	1.0718	−4.5%	62.9025
4	0.2635	0.2506	−4.9%	57.6243
5	0.9663	0.9663	0.0%	60.1482

Bi-Objective Space and Convergence

In figures 8 and 9 shows the bi-objective distribution, phase comparison, decision variable diversity, and Phase 1 convergence.

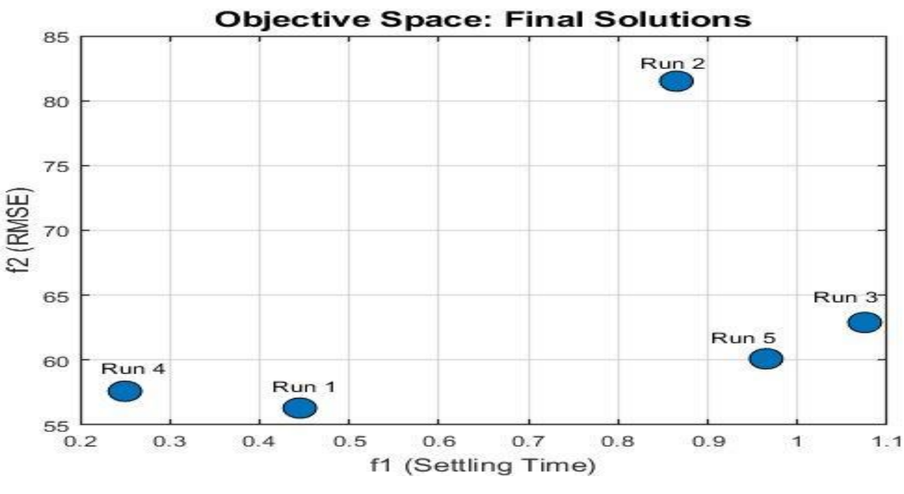


Figure 8. Bi-objective space: f_1 vs. f_2. run 4 dominates, closest to the ideal origin

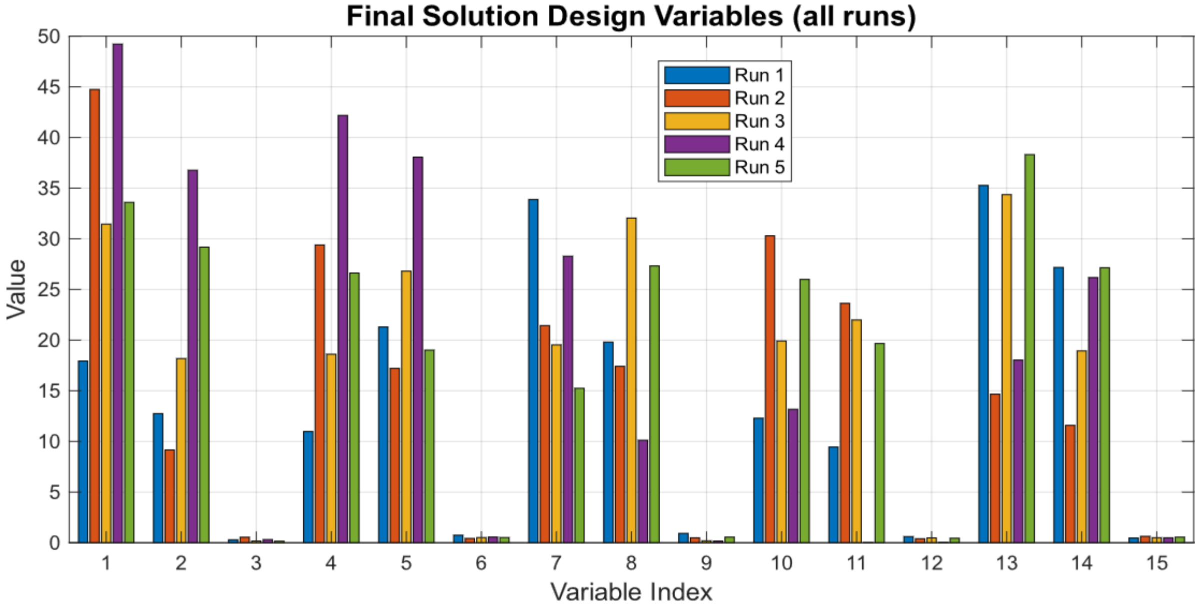


Figure 9. Distribution of optimized decision variables across 5 runs

Experiment III: Baseline Random Sampling

To justify guided search, 100 random FoPID configurations were evaluated using T_s/RMSE. Only 9% produced stable responses (∼7 h 39 min runtime), underscoring problem difficulty. The global best RMSE solution (Index 99, Figure 10) tracked the reference but settled slowly at 1.95 s; the best lexicographic solution under $f_1 \leq 1.2417$ s (Index 65, Figure 11) settled in 1.18 s but stabilised at ∼ 170 V with RMSE ≈ 89.13 V. Random sampling cannot navigate the speed–accuracy trade-off.

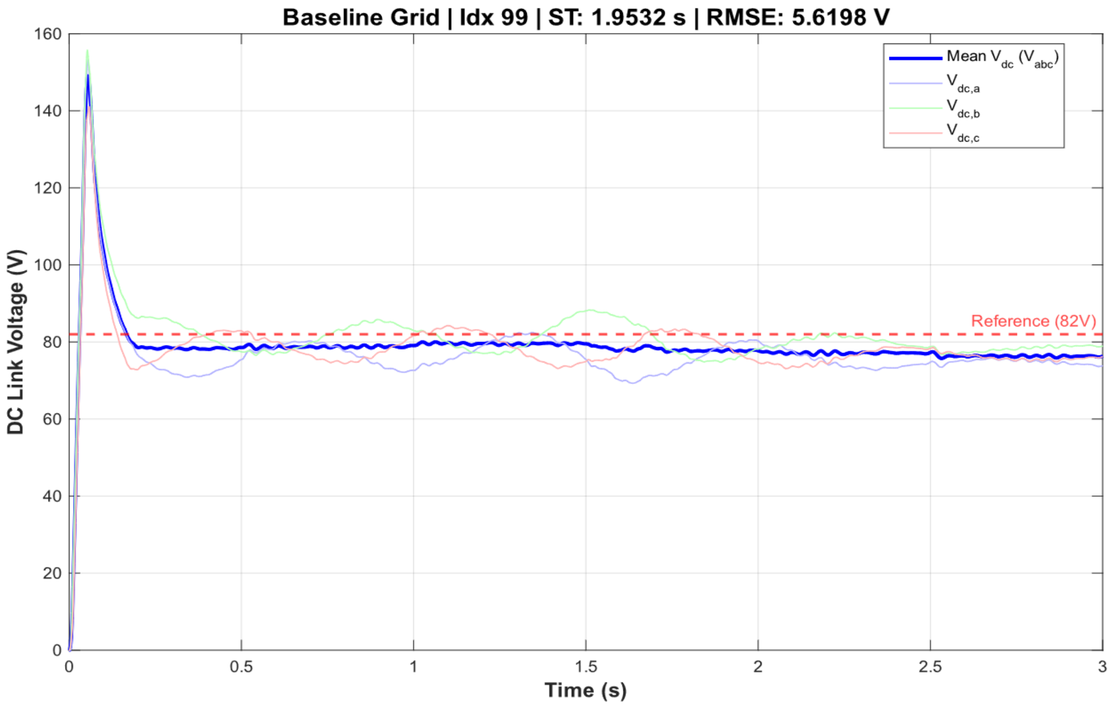


Figure 10. Baseline (index 99): accurate but sluggish (T_s ≈ 1.95 s)

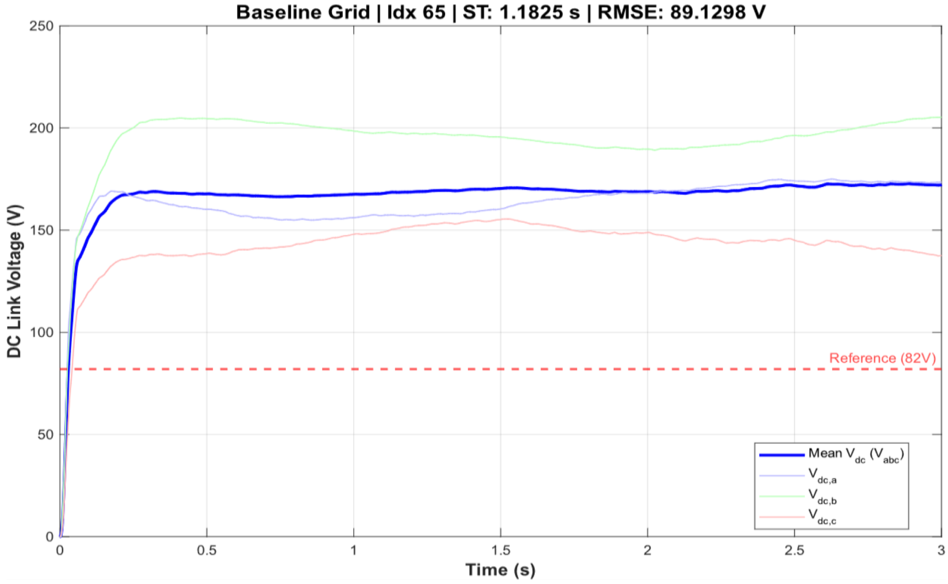


Figure 11. Baseline (index 65): fast ($T_s \approx 1.18$ s) but severe steady-state error (rmse ≈ 89.13 v)

In the ablation study, the effect of using various metaheuristic algorithms and metrics is determined on the tuning process of the FOPI-STATCOM. Lex-ITAE algorithms such as PSO, HHO, GWO display differences with regards to their mean costs and settling times, indicating the effect of the algorithm selection on transient behavior. Lex-PSO trials conducted in the order of priority of minimizing settling times present a relationship with RMSE, as demonstrated by Run 4 with 0.25 s and 57.6 V values for settling time and RMSE, respectively.

DISCUSSION

Objective formulation greatly affects not only the behavior of the algorithm itself but also the quality of control performance when tuning FOPI-STATCOM. The coupling between $f_1^{ITAE}$and f_2^{ITAE} . In the definition of Lex-ITAE, depending on the transient time $T_{Transitant}$ allows for the use of delay settling controllers to reduce steady-state assessment time and thus obtain artificially low costs. Lex-PSO reduces the dependence of settling time T_s on RMSE, but is vulnerable in a different way because optimization may lead the controller to operate at a very fast settling but offset point. In the case of Lex-ITAE, the convergence of Phase I turned out to be satisfactory enough, with the most efficient algorithm being HHO, while in Phase II, there appeared to be an inconsistency between numerical and actual control performance. PSO found the minimum ITAE cost under constraints, but caused oscillations at steady-state conditions.

Stagnation at a cost of 100 in the Lex-ITAE case study is the result of the hard constraint, rather than being caused by an error on the part of the algorithm, since the optimization of f_2^{ITAE} was hindered by slight violations of the f_1^{ITAE} . The Lex-PSO Run 4 managed to achieve T_sto = 0.2506 s with a Phase II refinement of 4.9%, while the DC link was stabilized at about 140 V, proving that stringent lexicographic ordering may meet mathematical requirements without necessarily resulting in physical feasibility. The passing of just one objective function f_1^* between phases fails to take into account the geometry of the decision space and runs the risk of local optimums trapping.

CONCLUSION AND FUTURE WORK

In this study, a lexicographic bi-objective approach for tuning FOPI controllers for grid-integrated STATCOMs was investigated using two proposed methods with a two-phase ε-constrained algorithmic structure, Lex-ITAE, that takes into account transient and steady-state ITAE subject to a hard penalty using five meta-heuristics, and Lex-PSO with a settling time and RMSE within a two-criteria fitness function with a normalized sacrifice metric. The effectiveness of both was tested for the purpose of optimization on an 11 kV cascaded H-bridge STATCOM in Simulink compared to a conventional random sampling method. Main results show the presence of a performance inversion with Lex-ITAE, whereby PSO obtained the minimum numerical value (0.56), while HHO, despite receiving the maximum number of penalties, yielded the smoothest tracking at 82 V as the ITAE integration window was vulnerable. Lex-PSO, despite being able to interpret T_s and RMSE values, showed that a strict settling requirement (0.25 s) caused a steady-state deviation of 140 V. The two approaches preferred numerical fitness over physical stability, punishing controllers whose performance was slightly out of the transient range. Random sampling showed that only 9% of configurations were stable, thus making meta-heuristic search appropriate. Regarding the comparison between approaches, Lex-ITAE combined with HHO resulted in better physical control, while Lex-PSO yielded easier-to-understand compromises but could be exploited. It can be concluded from the above discussion that even though the two-step, two-objective, and tolerance-based approach is logical, stricter lexicographic conditions need to be reconsidered for physical control. Future work will pursue: (i) adaptive constraint relaxation for smoother Phase I–II transitions, informed by the Lex-ITAE (hard rejection) and Lex-PSO (offset permission) failure modes; (ii) decision-space topology studies enabling intelligent initialization around Phase I optima under local L-smoothness; and (iii) coupled objectives that explicitly penalise steady-state offset alongside transient speed, preventing the exploitation observed in both formulations.

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